FOUR YEAR UNDERGRADUATE PROGRAMME IN MATHEMATICS

UNIVERSITY OF DELHI

DEPARTMENT OF MATHEMATICS

FOUR YEAR UNDERGRADUATE PROGRAMME

(Courses effective from Academic Year 2013-14)

SYLLABUS OF COURSES TO BE OFFERED

Discipline Courses I, Discipline Courses II

& Applied Courses

Note: The courses are uploaded as sent by the Department concerned. The scheme of marks will be determined by the University and will be corrected in the syllabus accordingly. Editing, typographical changes and formatting will be undertaken further.

Four Year Undergraduate Programme Secretariat
fouryearprog@gmail.com
### MATHEMATICS

**Teaching Hours:** Every semester, teaching will be spread over 16 weeks, including 2 weeks of review.

<table>
<thead>
<tr>
<th>Periods per week</th>
<th>Foundation Course</th>
<th>DC-I</th>
<th>DC-II</th>
<th>Applied Course</th>
<th>IMBH/NCC/NSS/Sport/CA</th>
</tr>
</thead>
<tbody>
<tr>
<td>For 14 weeks ever semester: Total 3 periods Lectures – 2 periods Class presentation – I period</td>
<td>For 14 weeks every semester: Total: 5 periods Lectures – 4 periods Class presentation – I periods. Practicals (wherever applicable) – 4 periods Tutorials* (wherever applicable) – as per requirement of course. In addition, two week for field work/project work/trip-related activity as required by the course curriculum.</td>
<td>For 14 weeks every semester Total: 5 periods Lectures -4 periods Class presentation – I period</td>
<td>For 16 weeks Total: 3 periods Practical/ hands-on experience/project work</td>
<td>For 16 weeks Total: 2 periods</td>
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</table>

| Maximun marks | Maximum 75 marks, with 40 marks for end semester examination and 35 marks for continuous evaluation of project work | Maximum 100 marks, with 75 marks for end semester examination and 25 marks for internal assessment. Where DC-I courses have a Practical component, these papers shall have maximum 150 marks, with 75 marks for end semester examination and 25 marks for internal assessment. | Maximum 100 marks, with 75 marks for end semester examination and 25 marks for internal assessment. Where DC-II courses have a Practical component, these papers shall have | Maximum 75 marks. Student will be continuously evaluated by the teacher(s) concerned. However, Applied Language Courses in the first year shall have an end semester examination of 40 marks and continuous | Not applicable |

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marks for internal assessment and 50 marks for the Practical (25 marks for continuous evaluation and 25 marks for end semester examination). The paper on research methodology (Semester 7) shall carry 100 marks. The Project that starts in semester 7 and continues in semester 8 shall carry 100 marks.

maximum 150 marks, with 75 marks for end semester examination and 25 marks for internal assessment and 50 marks for the Practical (25 marks for continuous evaluation and 25 marks for end semester examination)

evaluation of 35 marks

<table>
<thead>
<tr>
<th>Duration of end semester theory examination</th>
<th>2 hours</th>
<th>3 hours</th>
<th>3 hours</th>
<th>2 hours (only for Applied Language Courses)</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

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## Structures of DC-I, DC-II, AC Mathematics Courses

### DC-I STRUCTURE*

<table>
<thead>
<tr>
<th>Semester</th>
<th>No. of Papers</th>
<th>Papers</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>I.1 Calculus-I</td>
<td>17-19</td>
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<tr>
<td></td>
<td></td>
<td>I.2 Algebra-I</td>
<td>20-21</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>II.1 Analysis-I (Real Analysis)</td>
<td>22-23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II.2 Differential Equations-I</td>
<td>24-27</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>III.1 Analysis-II (Real Functions)</td>
<td>28-29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>III.2 Numerical Methods</td>
<td>30-33</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>IV.1 Calculus-II (Multivariate Calculus)</td>
<td>34-35</td>
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<td></td>
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<td>IV.2 Probability &amp; Statistics</td>
<td>36-40</td>
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<tr>
<td>5</td>
<td>3</td>
<td>V.1 Algebra-II (Group Theory-I)</td>
<td>41-42</td>
</tr>
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<td></td>
<td></td>
<td>V.2 Analysis-III (Riemann Integration &amp; Series of Functions)</td>
<td>43-44</td>
</tr>
<tr>
<td></td>
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<td>V.3 Diff. Eqns.-II (P.D.E. &amp; System of ODE)</td>
<td>45-48</td>
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<tr>
<td>6</td>
<td>3</td>
<td>VI.1 Algebra-III (Ring Theory &amp;</td>
<td>49-50</td>
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<tr>
<td>VI.2 Analysis-IV (Metric Spaces)</td>
<td>51-52</td>
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<tr>
<td>VI.3 Calculus of Variations &amp; Linear Programming</td>
<td>53-55</td>
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<tr>
<td>7</td>
<td>2+1</td>
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<tr>
<td>VII.1 Algebra-IV (Group Theory-II)</td>
<td>56</td>
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<tr>
<td>VII.2 Differential Equations-III</td>
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<tr>
<td>VII.3 Research</td>
<td>57-58</td>
<td>59</td>
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<tr>
<td>8</td>
<td>2+1</td>
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<tr>
<td>VIII.1 Analysis-V (Complex Analysis)</td>
<td>60-63</td>
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<tr>
<td>VIII.2 Algebra –V(Ring Theory &amp; Linear Algebra –II)</td>
<td>64-65</td>
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<tr>
<td>VIII.3 Research (Dissertation)</td>
<td>66</td>
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# DC-II STRUCTURE

<table>
<thead>
<tr>
<th>Semester</th>
<th>No. of Papers</th>
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<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>Calculus</td>
<td>69-70</td>
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<td>4</td>
<td>1</td>
<td>Linear Algebra</td>
<td>71-72</td>
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<tr>
<td>5</td>
<td>1</td>
<td>Differential Equations &amp; Mathematical Modeling</td>
<td>73-75</td>
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<td>6</td>
<td>1</td>
<td>Numerical Methods</td>
<td>76-78</td>
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<td>7</td>
<td>1</td>
<td>Real Analysis</td>
<td>79-81</td>
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<td>8</td>
<td>1</td>
<td>Abstract Algebra</td>
<td>82-83</td>
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## APPLIED COURSES STRUCTURE

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<tr>
<td>3</td>
<td>1</td>
<td>C++ Programming</td>
<td>85-88</td>
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<tr>
<td>4</td>
<td>1</td>
<td>Mathematical Finance</td>
<td>89-91</td>
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<tr>
<td>5</td>
<td>1</td>
<td>Cryptography and Network Systems</td>
<td>92-94</td>
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<tr>
<td>6</td>
<td>1</td>
<td>Discrete Mathematics</td>
<td>95-97</td>
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DISCIPLINE COURSES- I

MATHEMATICS

This course aims to create a solid foundation for assimilation of mathematical concepts and structures and build mathematical skills like creative, logical and analytical thinking. The syllabus has been designed to ensure that as the course progresses systematically, it provides a firm grounding in core mathematics subjects including Calculus, Analysis, Algebra, Differential Equations and Modelling real life problems. Tools such as Mathematica /SPSS/Maxima shall be used to enhance the understanding of fundamental mathematical concepts. The study of this course shall be of immense help to those who would like to pursue a career in fields like economics, physics, engineering, management science, computer science, operational research, mathematics and several others.
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| 7  | 2+1 | VII.1 Algebra-IV (Group Theory-II)  
|    |     | VII.2 Differential Equations-III  
|    |     | VII.3 Research                   | 56  |
| 8  | 2+1 | VIII.1 Analysis-V (Complex Analysis)  
|    |     | VIII.2 Algebra –V(Ring Theory & Linear Algebra –II)  
|    |     | VIII.3 Research (Dissertation)     | 60-63 |

*Each Practical will be of three classes. There is only one paper containing practicals in each semester. The practical paper may contain practicals from other papers of the same semester.
I.1 : Calculus I

Total marks: 150 (Theory: 75, Practical: 50, Internal Assessment: 25)

5 Periods (4 lectures + 1 students' presentation),
Practicals (4 periods per week per student),
Use of Scientific Calculators is allowed.

1st Week:
Hyperbolic functions, Higher order derivatives, Applications of Leibnitz rule.
[2]: Chapter 7 (Section 7.8)

2nd Week:
The first derivative test, concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test, limits at infinity, graphs with asymptotes.
[1]: Chapter 4 (Sections 4.3, 4.4)

3rd Week:
Graphs with asymptotes, L'Hopital's rule, applications in business, economics and life sciences.
[1]: Chapter 4 (Sections 4.5, 4.7)

4th Week:
Parametric representation of curves and tracing of parametric curves, Polar coordinates and tracing of curves in polar coordinates
[1]: Chapter 9 (Section 9.4)
[2]: Chapter 11 (Section 11.1)

5th Week:
Reduction formulae, derivations and illustrations of reduction formulae of the type
\[ \int \sin^n x \, dx, \int \cos^n x \, dx, \int \tan^n x \, dx, \int \sec^n x \, dx, \int (\log x)^n \, dx, \int \sin^n x \cos^m x \, dx \]
[2]: Chapter 8 (Sections 8.2-8.3, pages 532-538)

6th Week:
Volumes by slicing; disks and washers methods, Volumes by cylindrical shells.
[2]: Chapter 6 (Sections 6.2-6.3)

7th Week:
Arc length, arc length of parametric curves, Area of surface of revolution
[2]: Chapter 6 (Sections 6.4-6.5)
8th Week:
Techniques of sketching conics, reflection properties of conics
[2]: Chapter 11 (Section 11.4)

9th Week:
Rotation of axes and second degree equations, classification into conics using the discriminant
[2]: Chapter 11 (Section 11.5) (Statements of Theorems 11.5.1 and 11.5.2)

10th Week:
Introduction to vector functions and their graphs, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions.
[1]: Chapter 10 (Sections 10.1-10.2)

11th Week:
Modeling ballistics and planetary motion, Kepler’s second law.
[1]: Chapter 10 (Section 10.3)

12th Week:
Curvature, tangential and normal components of acceleration.
[1]: Chapter 10 (Section 10.4)
[2]: Chapter 13 (Section 13.5)

Practical / Lab work to be performed on a computer:
Modeling of the following problems using Matlab / Mathematica / Maple etc.

1. Plotting of graphs of function of type \( ax, a \in R \), \([x]\) (greatest integer function), \( \sqrt{ax + b} \), \( |ax + b| \), \( c \pm |ax + b| \), \( x^n \) (n even and odd positive integer), \( x^{-n} \) (n even and odd positive integer), \( \frac{1}{x^n} \) (n a positive integer) \( \frac{|x|}{x} \) for \( x \neq 0 \), \( \sin \frac{1}{x} \) for \( x \neq 0 \), \( x \sin \frac{1}{x} \) for \( x \neq 0 \), \( e^{\pm \frac{1}{x}} \), \( e^{ax+b} \), \( \log (ax + b) \), \( 1/(ax + b) \), \( \sin(ax + b) \), \( \cos(ax + b) \), \( |\sin(ax + b)| \), \( |\cos(ax + b)| \). Discuss the effect of \( a \) and \( b \) on the graph.
2. Plotting the graphs of polynomial of degree 4 and 5, the derivative graph, the second derivative graph and comparing them.
3. Sketching parametric curves.
4. Tracing of conics in Cartesian coordinates.
5. Obtaining surface of revolution of curves.
6. Sketching ellipsoid, hyperboloid of one and two sheets, elliptic cone, elliptic paraboloid, hyperbolic paraboloid using Cartesian co-ordinates.
7. To find numbers between two real numbers and plotting of finite and infinite subset of \( \mathbb{R} \).
8. Matrix operations (addition, multiplication, inverse, transpose, determinant, rank, eigenvectors, eigenvalues, Characteristic equation and verification of Cayley Hamilton equation, system of linear equations )
9. Graph of Hyperbolic functions.
11. Complex numbers and their representations, operations like addition, multiplication, division, modulus. Graphical representation of polar form.

REFERENCES:
I.2: Algebra I

**Total marks:** 100 (Theory: 75, Internal Assessment: 25)

**5 Periods** (4 lectures +1 students’ presentation),
**1 Tutorial** (per student per week)

(1<sup>st</sup> & 2<sup>nd</sup> Weeks)
Polar representation of complex numbers, nth roots of unity, De Moivre’s theorem for rational indices and its applications.

[1]: Chapter 2

(3<sup>rd</sup>, 4<sup>th</sup> & 5<sup>th</sup> Weeks)
Equivalence relations, Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set, Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

[2]: Chapter 2 (Section 2.4),
Chapter 3,
Chapter 4 (Sections 4.1 upto 4.1.6, 4.2 upto 4.2.11, 4.4(till 4.4.8), 4.3.7 to 4.3.9)
Chapter 5 (5.1.1, 5.1.4).

(6<sup>th</sup>, 7<sup>th</sup> & 8<sup>th</sup> Weeks)
Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation $Ax = b$, solution sets of linear systems, applications of linear systems, linear independence.

(9<sup>th</sup> & 10<sup>th</sup> Weeks)
Introduction to linear transformations, matrix of a linear transformation, inverse of a matrix, characterizations of invertible matrices.

(11<sup>th</sup> & 12<sup>th</sup> Weeks)
Subspaces of $R^n$, dimension of subspaces of $R^n$ and rank of a matrix, Eigen values, Eigen Vectors and Characteristic Equation of a matrix.

[3]: Chapter 1 (Sections 1.1-1.9),
Chapter 2 (Sections 2.1-2.3, 2.8-2.9),
Chapter 5 (Sections 5.1, 5.2).
REFERENCES:
II.1: Analysis I

Total marks: 100  (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students’ presentation),
1 Tutorial (per student per week)

1st Week
Review of Algebraic and Order Properties of $R$, $\delta$-neighborhood of a point in $R$, Idea of countable sets, uncountable sets and uncountability of $R$.
[1]: Chapter 1 (Section 1.3),
Chapter 2 (Sections 2.1, 2.2.7, 2.2.8)

2nd & 3rd Week
[1]: Chapter 2 (Sections 2.3, 2.4, 2.5.)

4th Week
Limit points of a set, Isolated points, Illustrations of Bolzano-Weierstrass theorem for sets.
[1]: Chapter 4 (Section 4.1)

5th Week
Sequences, Bounded sequence, Convergent sequence, Limit of a sequence.
[1]: Chapter 3 (Section 3.1)

6th & 7th Week
Limit Theorems, Monotone Sequences, Monotone Convergence Theorem.
[1]: Chapter 3 (Sections 3.2, 3.3)

8th & 9th Week
Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only), Bolzano Weierstrass Theorem for Sequences. Cauchy sequence, Cauchy’s Convergence Criterion.
[1]: Chapter 3 (Sections 3.4, 3.5)

10th, 11th & 12th Week
[2]: Chapter 6 (Section 6.2)
REFERENCES:

II.2 : Differential Equation-I

**Total marks:** 150  (Theory: 75, Internal Assessment: 25+ Practical: 50)

**5 Periods** (4 lectures +1 students’ presentation),
**Practicals** ( 4 periods per week per student)

(1st, 2nd & 3rd Weeks)
Differential equation and mathematical models. General, particular, explicit, implicit and singular solutions of a differential equation. Exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equation and Bernoulli equations, special integrating factors and transformations.
Ref.:
[2] Chapter 1 (section 1.1, 1.2, 1.4),
[3] Chapter 2 (section 2.1-2.4)

(4th, 5th & 6th Weeks)
Introduction to compartmental model, exponential decay model, lake pollution model (case study of Lake Burley Griffin), drug Assimilation into the blood (case of a single cold pill, case of a course of cold pills), exponential growth of population, limited growth of population, limited growth with harvesting.
Ref.:
[1]Chapter 2 (section 2.1, 2.2, 2.5-2.7),
Chapter 3 (section 3.1-3.3)

(7th, 8th & 9th Weeks)
General solution of homogeneous equation of second order, principle of super position for homogeneous equation, Wronskian: its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler’s equation, method of undetermined coefficients, method of variation of parameters.
Ref.:
[2] Chapter 3 (Section 3.1-3.3, 3.5)

(10th, 11th & 12th Weeks)
Equilibrium points, Interpretation of the phase plane, predatory-prey model and its analysis, epidemic model of influenza and its analysis, battle model and its analysis.
Ref.:
[1]CHAPTER 5 (SECTION 5.1-5.3, 5.7) CHAPTER 6 (SECTION 6.1-6.4)
REFERENCES:

LIST OF PRACTICALS
*(MODELLING OF FOLLOWING USING MATLAB/MATHEMATICA/MAPLE)*

1. Plotting of second order solution family of differential equation.
2. Plotting of third order solution family of differential equation.
3. Growth model (exponential case only).
4. Decay model (exponential case only).
5. Any two of the following:
   a) Lake pollution model (with constant/seasonal flow and pollution concentration).
   b) Case of single cold pill and a course of cold pills.
   c) Limited growth of population (with and without harvesting).
6. Any two of the following:
   a) Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   b) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   c) Battle model (basic battle model, jungle warfare, long range weapons).
7. Plotting of recursive sequences.
8. Find a value of $N$ that will make the following inequality holds for all $n > N$:
9. (i) $|\sqrt[3]{0.5} - 1| < 10^{-3}$, (ii) $|\sqrt[4]{n} - 1| < 10^{-3}$,
10. (iii) \((0.9)^n < 10^{-3}\), (iv) \(2^n/n! < 10^{-7}\) etc.

11. Study the convergence of sequences through plotting.

12. Verify Bolzano Weierstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.

13. Study the convergence/divergence of infinite series by plotting their sequences of partial sum.

14. Cauchy’s root test by plotting \(n^{th}\) roots.

15. Ratio test by plotting the ratio of \(n^{th}\) and \(n+1^{th}\) term.

16. For the following sequences \(<a_n>\), given \(\epsilon > 0\) and \(p \in \mathbb{N}\),

17. Find \(m \in \mathbb{N}\) such that 
   (i) \(|a_{m+p} - a_m| < \epsilon\),  
   (ii) \(|a_{2m+p} - a_{2m}| < \epsilon\)

18. 
   (a) 
   \[a_n = \frac{n+1}{n} \text{ for } (\epsilon = \frac{1}{2^k}, p = 10^j, j = 1,2,3,4,...), k = 0,1,2,5...\]
   
   (b) 
   \[a_n = \frac{1}{n} \text{ for } (\epsilon = \frac{1}{2^k}, p = 10^j, j = 1,2,3,4,...), k = 0,1,2,5...\]

   (c) 
   \[a_n = 1 + \frac{1}{2!} + ... + \frac{1}{n!} \text{ for } (\epsilon = \frac{1}{2^k}, p = 10^j, j = 1,2,3...), k = 0,1,2,...\]

   (d) 
   \[a_n = \frac{(-1)^n}{n} \text{ for } (\epsilon = \frac{1}{2^k}, p = 10^j, j = 1,2,3,...), k = 0,1,2,5...\]

   (e) 
   \[a_n = 1 - \frac{1}{2} + \frac{1}{3} - ... + \frac{(-1)^n}{n} \text{ for } (\epsilon = \frac{1}{2^k}, p = 10^j, j = 1,2,3...), k = 0,1,2,...\]

19. For the following series \(\sum a_n\), calculate
   
   (i) \(|a_{n+1}/a_n|\), (ii) \(|a_n|^{1/n}\), for \(n = 10^j, j = 1,2,3,...\), and identify the convergent series:
(a) $a_n = \left( \frac{1}{n} \right)^{1/n}$

(b) $a_n = \frac{1}{n}$

(c) $a_n = \frac{1}{n^2}$

(d) $a_n = \left( 1 + \frac{1}{\sqrt{n}} \right)^{-n^{3/2}}$

(e) $a_n = \frac{n!}{n^n}$

(f) $a_n = \frac{n^3 + 5}{3^n + 2}$

(g) $a_n = \frac{1}{n^2 + n}$

(h) $a_n = \frac{1}{\sqrt{n + 1}}$

(j) $a_n = \cos n$

(k) $a_n = \frac{1}{n \log n}$

(l) $a_n = \frac{1}{n (\log n)^2}$
III.1: Analysis II (Real Functions)

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students’ presentation),
1 Tutorial (per student per week)

1st Week
Limits of functions (ε-δ approach), sequential criterion for limits, divergence criteria.
   [1] Chapter 4, Section 4.1

2nd Week
Limit theorems, one sided limits.
   [1] Chapter 4, Section 4.2, Section 4.3 (4.3.1 to 4.3.4)

3rd Week
Infinite limits & limits at infinity.
   [1] Chapter 4, Section 4.3 (4.3.5 to 4.3.16)

4th Week
Continuous functions, sequential criterion for continuity & discontinuity.
   [1] Chapter 5, Section 5.1

5th Week
Algebra of continuous functions.
   [1] Chapter 5, Section 5.2

6th Week
Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem.

7th Week
Uniform continuity, non-uniform continuity criteria, uniform continuity theorem.
   [1] Chapter 5, Section 5.4 (5.4.1 to 5.4.3)

8th Week
Differentiability of a function at a point & in an interval, Carathéodory’s theorem, algebra of differentiable functions.
Relative extrema, interior extremum theorem. Rolle’s theorem, Mean value theorem, intermediate value property of derivatives - Darboux’s theorem. Applications of mean value theorem to inequalities & approximation of polynomials Taylor’s theorem to inequalities.

[1] Chapter 6, Section 6.2 (6.2.1 to 6.2.7, 6.2.11, 6.2.12)

Cauchy’s mean value theorem. Taylor’s theorem with Lagrange’s form of remainder, Taylor’s theorem with Cauchy’s form of remainder, application of Taylor’s theorem to convex functions, relative extrema. Taylor’s series & Maclaurin’s series expansions of exponential & trigonometric functions, \( \ln(1 + x), \frac{1}{(ax+b)} \) & \((1 + x)^n\).

[1] Chapter 6, Section 6.3 (6.3.2) Section 6.4 (6.4.1 to 6.4.6)

REFERENCES:
III.2: Numerical Methods

Total marks: 150  (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students’ presentation),
Practicals ( 4 periods per week per student)
Use of Scientific Calculators is allowed.

Algorithms, Convergence, Errors:
Relative, Absolute, Round off, Truncation.
   [1] 1.1, 1.2
   [2] 1.3 (pg 7-8)

Transcendental and Polynomial equations:
Bisection method, Newton’s method, Secant method. Rate of convergence of these methods.
   [2] 2.2, 2.3, 2.5, 2.10

System of linear algebraic equations:
   [2] 3.1, 3.2, 3.4

Interpolation:
Lagrange and Newton’s methods. Error bounds.
Finite difference operators. Gregory forward and backward difference interpolation.
   [2] 4.2, 4.3, 4.4

Numerical Integration:
Trapezoidal rule, Simpson’s rule, Simpsons 3/8th rule, Boole’s Rule.
Midpoint rule, Composite Trapezoidal rule, Composite Simpson’s rule.
   [1] 6.4, 6.5 (pg 467- 482)

Ordinary Differential Equations:
Euler’s method. Runge-Kutta methods of orders two and four.
   [1] 7.2 (pg 558 - 562), 7.4
REFERENCES:

SUGGESTED READING:

LIST OF PRACTICALS
Practical / Lab work to be performed on a computer: Use of computer aided software (CAS), for example Matlab / Mathematica / Maple / Maxima etc., for developing the following Numerical programs:
(i) Bisection Method
(ii) Secant Method
(iii) Newton Raphson Method
(iv) Gauss-Jacobi Method
(v) Gauss-Seidel Method
(vi) Lagrange Interpolation
(vii) Newton Interpolation
(viii) Composite Simpson’s Rule
(ix) Composite Trapezoidal Rule
(x) Euler’s Method
(xi) Runge Kutta Method of order 2 and 4.
(xii) Illustrations of the following:
   1. Let $f(x)$ be any function and $L$ be any number. For given $a$ and $\epsilon > 0$, find a $\delta > 0$ such that for all $x$ satisfying $0 < |x-a| < \delta$, the inequality $|f(x)-L| < \epsilon$ holds. For examples:
      (i) $f(x) = x+1, L = 5, a = 4, \epsilon = .01$
      (ii) $f(x) = \sqrt{x+1}, L = 1, a = 0, \epsilon = .1$
      (iii) $f(x) = x^2, L = 4, a = -2, \epsilon = .5$
(iv) \( f(x) = \frac{1}{x}, L = -1, a = -1, \epsilon = .1 \)

2. Discuss the limit of the following functions when \( x \) tends to 0:
\[
\pm \frac{1}{x}, \frac{1}{x} \cos \frac{1}{x}, x \sin \frac{1}{x}, x \cos \frac{1}{x}, x^2 \sin \frac{1}{x}, \frac{1}{x}, \frac{1}{x^2} \quad (n \in N), |x|, |x|, \frac{1}{x} - \sin x.
\]

3. Discuss the limit of the following functions when \( x \) tends to infinity:
\[
\frac{1}{e^x}, e^{-x}, \frac{1}{x} \sin \frac{1}{x}, \frac{1}{x} e^{-x}, \frac{1}{1+x}, x^2 \sin \frac{1}{x}, \frac{ax+b}{cx^2 + dx + e} \quad (a \neq 0 \neq c)
\]

4. Discuss the continuity of the functions at \( x = 0 \) in practical 2.

5. Illustrate the geometric meaning of Rolle’s theorem of the following functions on the given interval:
   (i) \( x^3 - 4x \) on \([-2, 2]\), (ii) \( (x-3)^4(x-5)^3 \) on \([3, 5]\) etc.

6. Illustrate the geometric meaning of Lagrange’s mean value theorem of the following functions on the given interval:
   (i) \( \log x \) on \([1/2, 2]\), (ii) \( x(x-1)(x-2) \) on \([0, 1/2]\),
   (iii) \( 2x^2 - 7x + 10 \) on \([2, 5]\) etc.

7. For the following functions and given \( \epsilon > 0 \), if exists, find \( \delta > 0 \) such that
\[
|f(x_j) - f(x_2)| < \epsilon \text{ whenever } |x_j - x_2| < \delta,
\]
and discuss uniformly continuity of the functions:
   (i) \( f(x) = \frac{1}{x} \) on \([0, 5]\), \( \epsilon = \frac{1}{2^j}, j = 0, 1, 2, 3, ... \)
   (ii) \( f(x) = \frac{1}{x} \) on \((0, 5]\), \( \epsilon = \frac{1}{2^j}, j = 0, 1, 2, 3, ... \)
   (iii) \( f(x) = x^2 \) on \([-1, 1]\), \( \epsilon = \frac{1}{2^j}, j = 0, 1, 2, 3, ... \)
   (iv) \( f(x) = \sin x \) on \((0, \infty)\), \( \epsilon = \frac{1}{2^j}, j = 0, 1, 2, 3, ... \)
   (v) \( f(x) = \sin x^2 \) on \((0, \infty)\), \( \epsilon = \frac{1}{2^j}, j = 0, 1, 2, 3, ... \)
(vi) \( f(x) = \frac{x}{1 + x^2} \) on \( \mathbb{R}, e = \frac{1}{2^j}, j = 0, 1, 2, 3, \ldots \)

(vii) \( f(x) = x^3 \) on \([0, 1], e = \frac{1}{2^j}, j = 0, 1, 2, 3, \ldots \)

8. Verification of Maximum – Minimum theorem, boundedness theorem & intermediate value theorem for various functions and the failure of the conclusion in case of any of the hypothesis is weekend.
9. Locating points of relative & absolute extremum for different functions
10. Relation of monotonicity & derivatives along with verification of first derivative test.
11. Taylor’s series - visualization by creating graphs:
   a. Verification of simple inequalities
   b. Taylor’s Polynomials – approximated up to certain degrees
   c. Convergence of Taylor’s series
   d. Non-existence of Taylor series for certain functions
   e. Convexity of the curves

Note: For any of the CAS Matlab / Mathematica / Maple / Maxima etc., the following should be introduced to the students.
Data types-simple data types, floating data types
arithmetic operators and operator precedence,
variables and constant declarations, expressions, input/output,
relational operators, logical operators and logical expressions,
control statements and loop statements, arrays.
IV.1: Calculus II (Multivariate Calculus)

Total marks: 100  (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)
Use of Scientific Calculators is allowed.

1st Week:
Functions of several variables, limit and continuity of functions of two variables
[1]: Chapter 11 (Sections 11.1(Pages 541-543), 11.2)

2nd Week:
Partial differentiation, total differentiability and differentiability, sufficient condition for differentiability.
[1]: Chapter 11 (Section 11.3, 11.4)

3rd Week:
Chain rule for one and two independent parameters, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes
[1]: Chapter 11 (Sections 11.5, 11.6)

4th Week:
Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems, Definition of vector field, divergence and curl
[1]: Chapter 11(Sections 11.7 (Pages 598-605), 11.8(Pages 610-614))
Chapter 13 (Pages 684-689)

5th Week:
Double integration over rectangular region, double integration over nonrectangular region
[1]: Chapter 12 (Sections 12.1, 12.2)

6th Week:
Double integrals in polar co-ordinates, Triple integrals, Triple integral over a parallelepiped and solid regions
[1]: Chapter 12 (Sections 12.3, 12.4 (Pages 652-655))

7th Week:
Volume by triple integrals, cylindrical and spherical co-ordinates.
[1]: Chapter 12 (Sections 12.4(Pages 656-660), 12.5)
8th Week:
Change of variables in double integrals and triple integrals.
   [1]: Chapter 12 (Section 12.6)

9th Week:
Line integrals, Applications of line integrals: Mass and Work.
   [1]: Chapter 13 (Section 13.2)

10th Week:
Fundamental theorem for line integrals, conservative vector fields, independence of path.
   [1]: Chapter 13 (Section 13.3)

11th Week:
Green's theorem, surface integrals, integrals over parametrically defined surfaces.
   [1]: Chapter 13 (Sections 13.4(Page 712–716), 13.5(Page 723–726, 729-730))

12th Week:
Stokes' theorem, The Divergence theorem.
   [1]: Chapter 13 (Section 13.6 (Page 733–737), 13.7 (Page 742–745)

REFERENCES:

SUGGESTED READING:
IV.2: PROBABILITY AND STATISTICS

Total marks: 150  (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students’ presentation),
Practicals (4 periods per week per student)

1st Week
Sample space, Probability axioms, Real random variables (discrete and continuous).

2nd Week
Cumulative distribution function, Probability mass/density functions, Mathematical expectation.

3rd Week
Moments, Moment generating function, Characteristic function.

4th Week
Discrete distributions: uniform, binomial, Poisson, Geometric, Negative Binomial distributions.

5th Week
Continuous distributions: Uniform, Normal, Exponential, Gamma distributions
[1]Chapter 1 (Section 1.1, .3, 1.5-1.9)
[2]Chapter 5 (Section 5.1-5.5,5.7), Chapter 6 (Sections 6.2-6.3,6.5-6.6)

6th Week
Joint cumulative distribution Function and its properties, Joint probability density functions – marginal and conditional distributions

7th Week
Expectation of a function of two random variables, Conditional expectations, Independent random variables, Covariance and correlation coefficient.

8th Week
Bivariate normal distribution, Joint moment generating function.

9th Week
Linear regression for two variables, The rank correlation coefficient.
[1] Chapter 2 (Section 2.1, 2.3-2.5)
[2] Chapter 4 (Exercise 4.47), Chapter 6 (Sections 6.7), Chapter 14 (Section 14.1, 14.2), Chapter 16 (Section 16.7)

10th Week
Chebyshev’s inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers.

11th Week
Central Limit Theorem for independent and identically distributed random variables with finite variance.

12th Week
Markov Chains, Chapman – Kolmogorov Equations, Classification of states.

REFERENCES:

PRACTICALS /LAB WORK TO BE PERFORMED ON A COMPUTER USING SPSS/EXCEL/Mathematica etc.

1. Calculation of
   (i) Arithmetic mean, geometric Mean, harmonic Mean
   (ii) Variance

2. Fitting of
   (i) Binomial Distribution
   (ii) Poisson Distribution
3. Calculation of
   (i) Correlation Coefficients
   (ii) Rank correlation

4. Fitting of polynomials
5. Regression curves

6. Draw the following surfaces and find level curves at the given heights:
   (i) \( f(x, y) = 10 - x^2 - y^2 \); \( z = 1, z = 6, z = 9 \),
   (ii) \( f(x, y) = x^2 + y^2 \); \( z = 1, z = 6, z = 9 \), (iii) \( f(x, y) = x^3 - y \); \( z = 1, z = 6 \),
   (iv) \( f(x, y) = x^2 + \frac{y^2}{4} \); \( z = 1, z = 5, z = 8 \),
   (v) \( f(x, y) = 4x^2 + y^2 \); \( z = 0, z = 1, z = 3, z = 5 \),
   (vi) \( f(x, y) = 2 - x - y \); \( z = -6, z = -4, z = -2, z = 0, z = 2, z = 4, z = 6 \).

7. Draw the following surfaces and discuss whether limit exits or not as \( (x, y) \) approaches to the given points. Find the limit, if it exists:
   (i) \( f(x, y) = \frac{x+y}{x-y} \); \( (x, y) \to (0,0) \) and \( (x, y) \to (1,3) \),
   (ii) \( f(x, y) = \frac{x-y}{\sqrt{x^2 + y^2}} \); \( (x, y) \to (0,0) \) and \( (x, y) \to (2,1) \),
   (iii) \( f(x, y) = (x+y)e^y \); \( (x, y) \to (1,1) \) and \( (x, y) \to (1,0) \),
   (iv) \( f(x, y) = e^y \); \( (x, y) \to (0,0) \) and \( (x, y) \to (1,0) \),
   (v) \( f(x, y) = \frac{x-y^2}{x^2 + y^2} \); \( (x, y) \to (0,0) \),
(vi) \( f(x, y) = \frac{x^2 + y}{x^2 + y^2}; \) \((x, y) \to (0,0),\)

(vii) \( f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}; \) \((x, y) \to (0,0)\) and \((x, y) \to (2,1),\)

(viii) \( f(x, y) = \frac{x^2 - y}{x + y}; \) \((x, y) \to (0,0)\) and \((x, y) \to (1,-1).\)

8. Draw the tangent plane to the following surfaces at the given point:

   (i) \( f(x, y) = \sqrt{x^2 + y^2} \) at \((3,1,\sqrt{10}),\) \((2,2,2),\)

   (ii) \( f(x, y) = 10 - x^2 - y^2 \) at \((0,0),\)

   (iii) \( x^2 + y^2 + z^2 = 9 \) at \((3,0,0),\)

   (iv) \( z = \arctan x \) at \((1,\sqrt{3},\pi/3)\) and \((2,2,\pi/4),\)

   (v) \( z = \log |x + y^2| \) at \((-3,-2,0).\)

9. Use an incremental approximation to estimate the following functions at the given point and compare it with the calculated value:

   (i) \( f(x, y) = 3x^4 + 2y^4 \) at \((1.01, 2.03),\) \((0.98, 1.03),\)

   (ii) \( f(x, y) = x^5 - 2y^3 \) at \((1.01, 0.98),\)

   (iii) \( f(x, y) = e^y \) at \((1.01, 0.98),\)

   (iv) \( f(x, y) = e^{x^2} \) at \((1.01, 0.98).\)

10. Find critical points and identify relative maxima, relative minima or saddle points to the following surfaces, if it exist:

   (i) \( z = x^2 + y^2,\) \( (ii) z = y^2 - x^2,\) \( (iii) z = 1 - x^2 - y^2,\) \( (iv) z = x^2y^4.\)

11. Draw the following regions \( D \) and check whether these regions are of Type I or Type II:

   (i) \( D = \{(x, y) | 0 \leq x \leq 2, 1 \leq y \leq e^x\},\)
(ii) \( D = \{(x, y) \mid \log y \leq x \leq 2, 1 \leq y \leq e^2\} \),

(iii) \( D = \{(x, y) \mid 0 \leq x \leq 1, x \leq y \leq 1\} \),

(iv) The region D is bounded by \( y = x^2 - 2 \) and the line \( y = x \),

(v) \( D = \{(x, y) \mid 0 \leq x \leq 1, x^3 \leq y \leq 1\} \),

(vi) \( D = \{(x, y) \mid 0 \leq x \leq y^3, 0 \leq y \leq 1\} \),

(vii) \( D = \{(x, y) \mid 0 \leq x \leq \frac{\pi}{4}, \cos x \leq y \leq \sin x\} \).
V.1: Algebra II (Group Theory I)

**Total marks:** 100 (Theory: 75, Internal Assessment: 25)

**5 Periods** (4 lectures + 1 students’ presentation),

**1 Tutorial** (per week per student)

**1st & 2nd Weeks**
Symmetries of a square, Dihedral groups, definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups.

**3rd Week**
Subgroups and examples of subgroups, centralizer, normalizer, center of a group, product of two subgroups.

**4th & 5th weeks**
Properties of cyclic groups, classification of subgroups of cyclic groups.

[1]: Chapters 1, Chapter 2, Chapter 3 (including Exercise 20 on page 66 and Exercise 2 on page 86), Chapter 4.

**6th, 7th & 8th Weeks**
Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, properties of cosets, Lagrange’s theorem and consequences including Fermat’s Little theorem.

**9th & 10th Weeks**
External direct product of a finite number of groups, normal subgroups, factor groups, Cauchy’s theorem for finite abelian groups.

[1]: Chapter 5 (till end of Theorem 5.7), Chapter 7 (till end of Theorem 7.2, including Exercises 6 and 7 on page 168), Chapter 8 (till the end of Example 2), Chapter 9 (till end of Example 10, Theorem 9.3 and 9.5).

**11th & 12th Weeks**
Group homomorphisms, properties of homomorphisms, Cayley’s theorem, properties of isomorphisms, First, Second and Third isomorphism theorems.

[1]: Chapter 6 (till end of Theorem 6.2), Chapter 10.
REFERENCES:

SUGGESTED READING:
V .2: ANALYSIS III (RIEMANN INTEGRATION & SERIES OF FUNCTIONS)

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students’ presentation),
1 Tutorial (per week per student)

1st Week
Riemann integration; inequalities of upper and lower sums; Riemann conditions of integrability.
[1] Chapter 6 (Art. 32.1 to 32.7)

2nd & 3rd Week
Riemann sum and definition of Riemann integral through Riemann sums; equivalence of two definitions; Riemann integrability of monotone and continuous functions, Properties of the Riemann integral; definition and integrability of piecewise continuous and monotone functions. Intermediate Value theorem for Integrals; Fundamental theorems of Calculus.
[1] Chapter 6 (Art. 32.8, 32.9, 33.1, 33.2, 33.3, 33.4 to 33.8, 33.9, 34.1, 34.3 )

4th Week
Improper integrals; Convergence of Beta and Gamma functions.
[3] Chapter 7 (Art. 7.8)

5th, 6th & 7th Week
Pointwise and uniform convergence of sequence of functions. Theorems on continuity, derivability and integrability of the limit function of a sequence of functions.
[2] Chapter 8, Section 8.1, Section 8.2 (8.2.1 – 8.2.2), Theorem 8.2.3, Theorem 8.2.4 and Theorem 8.2.5

8th & 9th Week
Series of functions; Theorems on the continuity and derivability of the sum function of a series of functions; Cauchy criterion for uniform convergence and Weierstrass M-Test
[2] Chapter 9, Section 9.4 (9.4.1 to 9.4.6)
10th, 11th & 12th Week
Limit superior and Limit inferior. Power series, radius of convergence, Cauchy Hadamard Theorem, Differentiation and integration of power series; Abel’s Theorem; Weierstrass Approximation Theorem.
[1] Chapter 4, Art. 26 (26.1 to 26.6), Theorem 27.5

REFERENCES:
3. Charles G. Denlinger, Elements of Real Analysis, Jones and Bartlett (Student Edition), 2011.
V.3: DIFFERENTIAL EQUATIONS-II (PDE & SYSTEM OF ODE)

Total marks: 150 (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students’ presentation),
Practicals (4 periods per week per student)

Section – 1 (1\textsuperscript{st}, 2\textsuperscript{nd} & 3\textsuperscript{rd} Weeks)
Partial Differential Equations – Basic concepts and Definitions, mathematical Problems.
First-Order Equations: Classification, Construction and Geometrical Interpretation.
Method of Characteristics for obtaining General Solution of Quasi Linear Equations.
[1]: Chapter 1: 1.2, 1.3
[1]: Chapter 2: 2.1-2.7

Section – 2 (3-weeks)
Derivation of heat equation, Wave equation and Laplace equation. Classification of second order linear equations as hyperbolic, parabolic or elliptic. Reduction of second order Linear Equations to canonical forms.
[1]: Chapter 3: 3.1, 3.2, 3.5, 3.6
[1]: Chapter 4: 4.1-4.5

Section – 3 (3-weeks)
[1]: Chapter 5: 5.1 – 5.5, 5.7
[1]: Chapter 7: 7.1, 7.2, 7.3, 7.5

Section -4 (3-weeks)
[2]: Chapter 7: 7.1, 7.3, 7.4,
REFERENCES:

LIST OF PRACTICALS
(MODELLING OF FOLLOWING USING MATLAB/MATHEMATICA/MAPLE)

1. Solution of Cauchy problem for first order PDE.
2. Plotting the characteristics for the first order PDE.
3. Plot the integral surfaces of a given first order PDE with initial data.

4. Solution of wave equation \( \frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0 \) for any 2 of the following associated conditions:

(a) \( u(x, 0) = \phi(x), \ u_t(x, 0) = \psi(x), \ x \in \mathbb{R}, \ t > 0. \)

(b) \( u(x, 0) = \phi(x), \ u_t(x, 0) = \psi(x), \ u(0, t) = 0, \ x \in (0, \infty), \ t > 0. \)

(c) \( u(x, 0) = \phi(x), \ u_t(x, 0) = \psi(x), \ u_x(0, t) = 0, \ x \in (0, \infty), \ t > 0. \)

(d) \( u(x, 0) = \phi(x), \ u_t(x, 0) = \psi(x), \ u(0, t) = 0, u(l, t) = 0, \ 0 < x < l, \ t > 0. \)
5. Solution of One-Dimensional heat equation $u_t = \kappa u_{xx}$, for a homogeneous rod of length $l$.

That is - solve the IBVP:

$$u_t = \kappa u_{xx}, \quad 0 < x < l, \quad t > 0,$$

$$u(0, t) = 0, u(l, t) = 0, t \geq 0,$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq l$$


7. Approximating solution to Initial Value Problems using any of the following approximate methods:

(a) The Euler Method

(b) The Modified Euler Method.

(c) The Runge-Kutta Method.

Comparison between exact and approximate results for any representative differential equation.

12. Draw the following sequence of functions on given the interval and discuss the pointwise convergence:

(i) $f_n(x) = x^n$ for $x \in \mathbb{R}$,

(ii) $f_n(x) = \frac{x}{n}$ for $x \in \mathbb{R}$,

(iii) $f_n(x) = \frac{x^2 + nx}{n}$ for $x \in \mathbb{R}$,

(iv) $f(x) = \frac{\sin nx + n}{n}$ for $x \in \mathbb{R}$

(v) $f_n(x) = \frac{x}{x + n}$ for $x \in \mathbb{R}$, $x \geq 0$,

(vi) $f_n(x) = \frac{nx}{1 + n^2 x^2}$ for $x \in \mathbb{R}$,
\[(\text{Vii}) \quad f_n(x) = \frac{nx}{1+nx} \quad \text{for} \ x \in \mathbb{R}, \ x \geq 0, \]

\[(\text{viii}) \quad f_n(x) = \frac{x^n}{1+x^n} \quad \text{for} \ x \in \mathbb{R}, \ x \geq 0 \]

13. Discuss the uniform convergence of sequence of functions above.
VI .1: ALGEBRA III (RINGS AND LINEAR ALGEBRA I)

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students' presentation),
1 Tutorial (per week per student)

(1ˢᵗ & 2ⁿᵈ Weeks)
Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring.

(3ʳᵈ & 4ᵗʰ Weeks)
Ideals, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals.

(5ᵗʰ & 6ᵗʰ Weeks)
Ring homomorphisms, properties of ring homomorphisms, Isomorphism theorems I, II and III, field of quotients.
[2]: Chapter 12, Chapter 13, Chapter 14, Chapter 15.

(7ᵗʰ & 8ᵗʰ Weeks)
Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces.

(9ᵗʰ & 10ᵗʰ Weeks)
Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations.

(11ᵗʰ & 12ᵗʰ Weeks)
Isomorphisms, Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix.
[1]: Chapter 1 (Sections 1.2-1.6, Exercise 29, 33, 34, 35), Chapter 2 (Sections 2.1-2.5).

REFERENCES:
SUGGESTED READING:
1. S Lang, Introduction to Linear Algebra (2nd edition), Springer, 2005
VI.2: ANALYSIS IV (METRIC SPACES)

**Total marks:** 100 (Theory: 75, Internal Assessment: 25)
**5 Periods** (4 lectures +1 students’ presentation),
**1 Tutorial** (per week per student)

**(1st Week)**
Metric spaces: definition and examples.
[1] Chapter1, Section 1.2 (1.2.1 to 1.2.6).

**(2nd Week)**
Sequences in metric spaces, Cauchy sequences.
[1] Chapter1, Section 1.3, Section 1.4 (1.4.1 to 1.4.4)

**(3rd Week)**
Complete Metric Spaces.
[1] Chapter1, Section 1.4 (1.4.5 to 1.4.14 (ii)).

**(4th Week)**
Open and closed balls, neighbourhood, open set, interior of a set
[1] Chapter2, Section 2.1 (2.1.1 to 2.1.16)

**(5th & 6th Weeks)**
Limit point of a set, closed set, diameter of a set, Cantor’s Theorem.
[1] Chapter2, Section 2.1 (2.1.17 to 2.1.44)

**(7th Week)**
Subspaces, dense sets, separable spaces.
[1] Chapter2, Section 2.2, Section 2.3 (2.3.12 to 2.3.16)

**(8th Week)**
Continuous mappings, sequential criterion and other characterizations of continuity.
[1] Chapter3, Section 3.1

**(9th Week)**
Uniform continuity
[1] Chapter3, Section 3.4 (3.4.1 to 3.4.8)
(10th Week)
Homeomorphism, Contraction mappings, Banach Fixed point Theorem.
[1] Chapter3, Section 3.5 (3.5.1 to 3.5.7(iv) ), Section 3.7 (3.7.1 to 3.7.5)

(11th Week)
Connectedness, connected subsets of R, connectedness and continuous mappings.
[1] Chapter4, Section 4.1 (4.1.1 to 4.1.12)

(12th Week)
Compactness, compactness and boundedness, continuous functions on compact spaces.
[1] Chapter5, Section 5.1 (5.1.1 to 5.1.6), Section 5.3 (5.3.1 to 5.3.11)

REFERENCES:

SUGGESTED READINGS:
VI.3: CALCULUS OF VARIATIONS AND LINEAR PROGRAMMING

**Total marks:** 150 (Theory: 75, Internal Assessment: 25, Practical: 50)

**4 Lectures** + 4 Practicals + 1 Presentation

*(1st & 2nd Week)*

[1]: Chapter 1 (Sections 1, 3, 4 and 6).

*(3rd & 4th Week)*
Introduction to linear programming problem, Graphical method of solution, Basic feasible solutions, Linear programming and Convexity.

[2]: Chapter 2 (Section 2.2), Chapter 3 (Sections 3.1, 3.2 and 3.9).

*(5th & 6th Week)*
Introduction to the simplex method, Theory of the simplex method, Optimality and Unboundedness.

[2]: Chapter 3 (Sections 3.3 and 3.4).

*(7th & 8th Week)*
The simplex tableau and examples, Artificial variables.

[2]: Chapter 3 (Sections 3.5 and 3.6).

*(9th & 10th Week)*

[2]: Chapter 4 (Sections 4.1, 4.2, 4.4 and 4.5).

*(11th & 12th Week)*

[3]: Chapter 5 (Sections 5.1, 5.3 and 5.4)
PRACTICAL/LAB WORK TO BE PERFORMED ON A COMPUTER:
(MODELLING OF THE FOLLOWING PROBLEMS USING EXCEL
SOLVER/LINGO/MATHEMATICA, ETC.)

(i) Formulating and solving linear programming models on a spreadsheet using excel solver.
[2]: Appendix E and Chapter 3 (Examples 3.10.1 and 3.10.2).
[4]: Chapter 3 (Section 3.5 with Exercises 3.5-2 to 3.5-5)

(ii) Finding solution by solving its dual using excel solver and giving an interpretation of the
dual.
[2]: Chapter 4 (Examples 4.3.1 and 4.4.2)

(iii) Using the excel solver table to find allowable range for each objective function coefficient,
and the allowable range for each right-hand side.
[4]: Chapter 6 (Exercises 6.8-1 to 6.8-5).

(iv) Formulating and solving transportation and assignment models on a spreadsheet using
solver.
[4]: Chapter 8 (CASE 8.1: Shipping Wood to Market, CASE 8.3: Project Pickings).

From the Metric space paper, exercises similar to those given below:

1. Calculate $d(x,y)$ for the following metrics

   (i) $X=\mathbb{R}$, $d(x,y)=|x-y|$
   (ii) $X=\mathbb{R}^3$, $d(x,y)=\left(\sum (x_i-y_i)^2\right)^{1/2}$

   \[ \begin{align*}
   x & : 0, 1, \pi, e \\
   y & : 1, 2, \sqrt{2}
   \end{align*} \]

   x: (0,1,-1), (1,2,\pi), (2,-3,5)
   y: (1, 2, .5), (e,2,4), (-2,-3,5)

   (iii) $X=C[0,1]$, $d(f,g)=\sup |f(x)-g(x)|$

   f(x): $x^2$, $\sin x$, $\tan x$
   g(x): $x$, $|x|$, $\cos x$

2. Draw open balls of the above metrics with centre and radius of your choice.

3. Find the fixed points for the following functions

   f(x)=x^2 , g(x)=\sin x, h(x)=\cos x in X=[-1, 1],
   f(x,y)=(\sin x, \cos y), g(x,y)=(x^2, y^2) in X=\{(x,y): x^2+y^2\leq 1\},

under the Euclidean metrics on $\mathbb{R}$ and $\mathbb{R}^2$ respectively.
4. Determine the compactness and connectedness by drawing sets in $\mathbb{R}^2$.

REFERENCES:


SUGGESTED READING:


VII.1: ALGEBRA IV (GROUP THEORY II)

Total marks: 100 (Theory: 75, Internal Assessment: 25)
5 Periods (4 lectures +1 students’ presentation),
1 Tutorial (per week per student)

(1st, 2nd & 3rd Weeks)
Automorphism, inner automorphism, automorphism groups, automorphism groups of
finite and infinite cyclic groups, applications of factor groups to automorphism groups,
Characteristic subgroups, Commutator subgroup and its properties.
[1]: Chapter 6, Chapter 9 (Theorem 9.4), Exercises 1-4 on page 168, Exercises 52, 58
on page Pg 188.

(4th, 5th & 6th Weeks)
Properties of external direct products, the group of units modulo n as an external
direct product, internal direct products, Fundamental Theorem of finite abelian groups.
[1]: Chapter 8, Chapter 9 (Section on internal direct products), Chapter 11.

(7th, 8th & 9th Weeks)
Group actions, stabilizers and kernels, permutation representation associated with a
given group action, Applications of group actions: Generalized Cayley’s theorem, Index
theorem.

(10th, 11th & 12th Weeks)
Groups acting on themselves by conjugation, class equation and consequences,
conjugacy in $S_n$, $p$-groups, Sylow’s theorems and consequences, Cauchy’s theorem,
Simplicity of $A_n$ for $n \geq 5$, non-simplicity tests.
[2]: Chapter 1 (Section 1.7), Chapter 2 (Section 2.2), Chapter 4 (Section 4.1-4.3,
4.5-4.6).
[1]: Chapter 25.

REFERENCES:
Sons (Asia) Pvt. Ltd, Singapore, 2004
VII.2: DIFFERENTIAL EQUATIONS-III (TRANSFORMS & BOUNDARY VALUE PROBLEMS)

Total marks: 150  (Theory: 75, Internal Assessment: 25+ Practical: 50)
5 Periods (4 lectures +1 students' presentation),
Practicals ( 4 periods per week per student)

(1\textsuperscript{st}, 2\textsuperscript{nd} & 3\textsuperscript{rd} Weeks)
Introduction: power series solution methods, series solutions near ordinary point, series solution about regular singular point. Special functions: Bessel's equation and function, Legendre’s equation and function.
  [1] chapter 8: 8.1-8.3
  [2]chapter 8: 8.6, 8.9

(4\textsuperscript{th}, 5\textsuperscript{th} & 6\textsuperscript{th} Weeks)
Sturm Theory: Self-Ad joint equation of the second order, Abel's formula, Sturm separation and comparison theorems, method of Separation of variables: The Laplace and beam equations, non-homogeneous equation.
  [3] Chapter 11: 11.8
  [2] Chapter 7: 7.7, 7.8

(7\textsuperscript{th}, 8\textsuperscript{th} & 9\textsuperscript{th} Weeks)
Boundary Value Problem: Introduction, maximum and minimum Principal, Uniqueness and continuity theorem, Dirichlet problem for a circle, Neumann Problem for a circle.

(10\textsuperscript{th}, 11\textsuperscript{th} & 12\textsuperscript{th} Weeks)
Integral transform-introduction, Fourier transforms, properties of Fourier transforms, convolution Theorem of Fourier transforms, Laplace transforms, properties of Laplace transforms, convolution theorem of Laplace transforms
REFERENCES:

LIST OF PRACTICALS FOR DE-III
(MODELLING OF FOLLOWING USING MATLAB/MATHEMATICA/MAPLE)

1. Plotting of [0,1]. Legendre polynomial for n=1 to 5 in the interval Verifying graphically that all roots of $P_n(x)$ lie in the interval [0,1].
2. Automatic computation of coefficients in the series solution near ordinary points.
3. Plotting of the Bessel’s function of first kind of order 0 to 3.
4. Automating the frobenius Series method.
5. Use of Laplace transforms to plot the solutions of
   a. Massspring systems with and without external forces and draw inferences.
   b. LCR circuits with applied voltage. Plot the graph of current and charge w.r.t. time and draw inferences.
6. Find Fourier series of different functions. Plot the graphs for $n = 1-6$. Draw inferences for the solutions as $n$ tends to infinity.
7. Finding and Plotting Laplace transforms of various functions and solving a differential equation using Laplace transform.
9. Finding and Plotting the convolution of 2 functions and verify the Convolution theorem of the Fourier Transform/Laplace Transform.
10. Solve the Laplace equation describing the steady state temperature distribution in a thin rectangular slab, the problem being written as:
    \[ \nabla^2 u = 0, \quad 0 < x < a, 0 < y < b, \]
    \[ u(x, 0) = f(x), \quad 0 \leq x \leq a, \]
    \[ u(x, b) = 0, \quad 0 \leq x \leq a, \]
    \[ u_x(0, y) = 0, \quad u_x(a, y) = 0, \]
    for prescribed values of $a$ and $b$, and given function $f(x)$. 
VII.3 Research Methodology

Study and Practice of Modern Mathematics (3-weeks)

Studying mathematics; writing homework assignments and problem solving; writing mathematics; mathematical research; presenting mathematics; professional steps.

[1] Chapter 1-6

LaTeX and HTML (4-weeks)

Elements of LaTeX; Hands-on training of LaTeX; graphics in LaTeX; PSTricks; Beamer presentation; HTML, creating simple web pages, images and links, design of web pages.

[1] Chapter 9-11, 15

Computer Algebra Systems and Related Softwares (5-weeks)

Use of Mathematica, Maple, and Maxima as calculator, in computing functions, in making graphs; MATLAB/Octave for exploring linear algebra and to plot curve and surfaces; the statistical software R: R as a calculator, explore data and relations, testing hypotheses, generate table values and simulate data, plotting.

[1] Chapter 12-14

References:


List of Practical for Research Methodology in Mathematics

(2L per week per student)

Twelve practical should be done by each student. The teacher can assign practical from the following list. These are exercises from [1].

1. Exercise 4-9 (pages 73-76) (2 practical)
2. Exercise 1,3,5-9,11-13 (pages 88-90) (2 practical)
3. Exercise in page 95 (1 practical)
4. Exercise 5, 10, 11 (1 practical)
5. Exercise 1-4, 8-12 (pages 125-128) (4 practical)
6. Exercise 4, 6-10 (pages 146-147) (1 practical)
7. Exercise 1-4 (page 161) (1 practical)
VIII.1: ANALYSIS V (COMPLEX ANALYSIS)

**Total marks:** 150 (Theory: 75, Practical: 50, Internal Assessment: 25)

**5 Periods** (4 lectures +1 students’ presentation),

**Practical** (4 periods per week per student),

**(1\textsuperscript{st} & 2\textsuperscript{nd} Week):**

Limits, Limits involving the point at infinity, continuity.
Properties of complex numbers, regions in the complex plane, functions of complex variable, mappings. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.

[1]: Chapter 1 (Section 11), Chapter 2 (Section 12, 13) Chapter 2 (Sections 15, 16, 17, 18, 19, 20, 21, 22)

**(3\textsuperscript{rd}, 4\textsuperscript{th} & 5\textsuperscript{th} Week):**

Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, derivatives of functions, definite integrals of functions.

[1]: Chapter 2 (Sections 24, 25), Chapter 3 (Sections 29, 30, 34), Chapter 4 (Section 37, 38)

**(6\textsuperscript{th} Week):**

Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals.

[1]: Chapter 4 (Section 39, 40, 41, 43)

**(7\textsuperscript{th} Week):**

Antiderivatives, proof of antiderivative theorem, Cauchy-Goursat theorem, Cauchy integral formula.

[1]: Chapter 4 (Sections 44, 45, 46, 50)

**(8\textsuperscript{th} Week):**

An extension of Cauchy integral formula, consequences of Cauchy integral formula, Liouville’s theorem and the fundamental theorem of algebra.

[1]: Chapter 4 (Sections 51, 52, 53)

**(9\textsuperscript{th} Week):**

Convergence of sequences and series, Taylor series and its examples.

[1]: Chapter 5 (Sections 55, 56, 57, 58, 59)

**(10\textsuperscript{th} Week):**
Laurent series and its examples, absolute and uniform convergence of power series, uniqueness of series representations of power series.

[1]: Chapter 5 (Sections 60, 62, 63, 66)

(11th Week):
Isolated singular points, residues, Cauchy’s residue theorem, residue at infinity.

[1]: Chapter 6 (Sections 68, 69, 70, 71)

(12th Week):
Types of isolated singular points, residues at poles and its examples, definite integrals involving sines and cosines.

[1]: Chapter 6 (Sections 72, 73, 74), Chapter 7 (Section 85).

REFERENCES:

SUGGESTED READING:

LAB WORK TO BE PERFORMED ON A COMPUTER
(MODELING OF THE FOLLOWING PROBLEMS USING MATLAB/ MATHEMATICA/ MAPLE ETC.)

1. Declaring a complex number and graphical representation.
   e.g. $Z_1 = 3 + 4i, Z_2 = 4 - 7i$

2. Program to discuss the algebra of complex numbers.
   e.g., if $Z_1 = 3 + 4i, Z_2 = 4 - 7i$, then find $Z_1 + Z_2, Z_1 - Z_2, Z_1 * Z_2, and Z_1 / Z_2$

3. To find conjugate, modulus and phase angle of an array of complex numbers.
e.g., $Z = [2+3i\ 4-2i\ 6+11i\ 2-5i]$

4. To compute the integral over a straight line path between the two specified end points.
   
   e.g., $\int_C \sin Z\,dz$, where $C$ is the straight line path from $-1+i$ to $2-i$.

5. To perform contour integration.
   
   e.g., (i) $\int_C (Z^2 - 2Z + 1)\,dz$, where $C$ is the contour given by $x = y^2 + 1$; $-2 \leq y \leq 2$.
   
   (ii) $\int_C (Z^3 + 2Z^2 + 1)\,dz$, where $C$ is the contour given by $x^2 + y^2 = 1$, which can be parameterized by $x = \cos(t)$, $y = \sin(t)$ for $0 \leq y \leq 2\pi$.

6. To plot the complex functions and analyze the graph.
   
   e.g., (i) $f(z) = Z$
   
   (ii) $f(z) = Z^2$
   
   (iii) $f(z) = (Z^4 - 1)^{1/4}$
   
   (iv) $f(z) = \overline{z}$, $f(z) = iz$, $f(z) = z^2$, $f(z) = e^z$ etc.

7. To perform the Taylor series expansion of a given function $f(z)$ around a given point $z$.
   
   The number of terms that should be used in the Taylor series expansion is given for each function. Hence plot the magnitude of the function and magnitude of its Taylor series expansion.
   
   e.g., (i) $f(z) = \exp(z)$ around $z = 0$, $n = 40$.
   
   (ii) $f(z) = \exp(z^2)$ around $z = 0$, $n = 160$.

8. To determine how many terms should be used in the Taylor series expansion of a given function $f(z)$ around $z = 0$ for a specific value of $z$ to get a percentage error of less than 5%.
e.g., For \( f(z) = \exp(z) \) around \( z = 0 \), execute and determine the number of necessary terms to get a percentage error of less than 5% for the following values of \( z \):

(i) \( z = 30 + 30 \, i \)

(ii) \( z = 10 + 10\sqrt{3} \, i \)

9. To perform Laurents series expansion of a given function \( f(z) \) around a given point \( z \).

   e.g., (i) \( f(z) = (\sin z - 1)/z^4 \) around \( z = 0 \)

   (ii) \( f(z) = \cot(z)/z^4 \) around \( z = 0 \).

10. To compute the poles and corresponding residues of complex functions.

   e.g., \( f(z) = \frac{z + 1}{z^3 - 2z + 2} \)

12. To perform Conformal Mapping and Bilinear Transformations.
VIII.2: ALGEBRA V (RINGS AND LINEAR ALGEBRA II)

**Total marks:** 100 (Theory: 75, Internal Assessment: 25)

**5 Periods** (4 lectures + 1 students’ presentation),

**1 Tutorial** (per week per student)

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**1\(^{st}\), 2\(^{nd}\), 3\(^{rd}\) & 4\(^{th}\) Weeks**

Polynomial rings over commutative rings, division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, unique factorization in \(\mathbb{Z}[x]\).

Divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains.

[1]: Chapter 16, Chapter 17, Chapter 18.

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**5\(^{th}\), 6\(^{th}\), 7\(^{th}\) & 8\(^{th}\) Weeks**

Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators, Eigenspaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator.

[2]: Chapter 2 (Section 2.6), Chapter 5 (Sections 5.1-5.2, 5.4), Chapter 7 (Section 7.3).

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**9\(^{th}\), 10\(^{th}\), 11\(^{th}\) & 12\(^{th}\) Weeks**

Inner product spaces and norms, Gram-Schmidt orthogonalisation process, orthogonal complements, Bessel’s inequality, the adjoint of a linear operator, Least Squares Approximation, minimal solutions to systems of linear equations, Normal and self-adjoint operators, Orthogonal projections and Spectral theorem.

[2]: Chapter 6 (Sections 6.1-6.4, 6.6).
REFERENCES:

SUGGESTED READING:

(Linear Algebra)

(Ring theory and group theory)
Discipline Courses - II
Mathematics

The course structure of Discipline-II in MATHEMATICS is a blend of pure and applied papers. This study of this course would be beneficial to students belonging to variety of disciplines such as economics, physics, engineering, management sciences, computer sciences, operational research and natural sciences. The course has been designed to help one pursue a masters degree in mathematics. The first two courses on Calculus and Linear Algebra are central to both pure and applied mathematics. The next two courses with practical components are of applied nature. The course on Differential Equations and Mathematical Modeling deals with modeling of much physical, technical, or biological process in the form of differential equations and their solution procedures. The course on Numerical Methods involves the design and analysis of techniques to give approximate but accurate solutions of hard problems using iterative methods. The last two courses on Real Analysis and Abstract Algebra provides an introduction to the two branches of pure mathematics in a rigorous and definite form.
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CALCULUS (SEMESTER III)

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
4 Lectures, 1 Presentation

(1\textsuperscript{st} Week)
$\varepsilon$-$\delta$ Definition of limit of a function, One sided limit, Limits at infinity, Horizontal asymptotes
   Sections 2.3, 2.4 [1]

(2\textsuperscript{nd} Week)
Infinite limits, Vertical asymptotes, Linearization, Differential of a function
   Sections 2.5, 3.8 [1]

(3\textsuperscript{rd} Week)
Concavity, Points of inflection, Curve sketching
   Sections 4.4 [1]

(4\textsuperscript{th} Week)
Indeterminate forms: L'Hôpital's rule, Volumes by slicing, Volumes of solids of revolution by the disk method
   Sections 4.6, 6.1 (Pages 396 to 402) [1]

(5\textsuperscript{th} Week)
Volumes of solids of revolution by the washer method, Volume by cylindrical shells, Length of plane curves
   Sections 6.1 (Pages 403 to 405), 6.2, 6.3 [1]

(6\textsuperscript{th} Week)
Area of surface of revolution, Improper integration: Type I and II, Tests of convergence and divergence
   Sections 6.5 (Pages 436 to 442), 8.8 [1]

(7\textsuperscript{th} Week)
Polar coordinates, Graphing in polar coordinates
   Sections 10.5, 10.6 [1]
(8th Week)
Vector valued functions: Limit, Continuity, Derivatives, Integrals, Arc length, Unit tangent vector
   Sections 13.1, 13.3 [1]

(9th Week)
Curvature, Unit normal vector, Torsion, Unit binormal vector, Functions of several variables: Graph, Level curves
   Sections 13.4, 13.5, 14.1 [1]

(10th Week)
Limit, Continuity, Partial derivatives, Differentiability
   Sections 14.2, 14.3 [1]

(11th Week)
Chain Rule, Directional derivatives, Gradient
   Sections 14.4, 14.5 [1]

(12th Week)
Tangent plane and normal line, Extreme values, Saddle points
   Section 14.6 (Pages 1015 to 1017), 14.7 [1]

REFERENCES:

SUGGESTED READING:

Note: The emphasis is on learning of methods/techniques of calculus and on the application of these methods for solving variety of problems.
LINEAR ALGEBRA (SEMESTER IV)

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
4 Lectures, 1 Presentation

(1st Week)
Fundamental operation with vectors in Euclidean space $\mathbb{R}^n$, Linear combination of vectors, Dot product and their properties, Cauchy–Schwarz inequality, Triangle inequality, Projection vectors
Sections 1.1, 1.2 [1]

(2nd Week)
Some elementary results on vector in $\mathbb{R}^n$, Matrices: Gauss–Jordan row reduction, Reduced row echelon form, Row equivalence, Rank
Sections 1.3 (Pages 31 to 40), 2.2 (Pages 98 to 104), 2.3 (Pages 110 to 114, Statement of Theorem 2.3) [1]

(3rd Week)
Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices
Sections 2.3 (Pages 114 to 121, Statements of Lemma 2.7 and Theorem 2.8), 3.4 [1]

(4th Week)
Definition and examples of vector space, Some elementary properties of vector spaces, Subspace
Sections 4.1, 4.2 (Statement of Theorem 4.3) [1]

(5th Week)
Span of a set, A spanning set for an eigenspace, Linear independence and linear dependence of vectors
Sections 4.3 (Statement of Theorem 4.5), 4.4 [1]

(6th Week)
Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets, Application of rank: Homogenous and nonhomogenous systems of equations
Section 4.5 (Statements of Lemma 4.11 and Theorem 4.13) [1]
Section 6.6 (Pages 289 to 291) [2]

(7th Week)
Coordinates of a vector in ordered basis, Transition matrix, Linear transformations:
Definition and examples, Elementary properties
Section 6.7 (Statement of Theorem 6.15) [2]
Section 5.1 (Statements of Theorem 5.2 and Theorem 5.3) [1]

(8th Week)
The matrix of a linear transformation, Linear operator and Similarity
Section 5.2 (Statements of Theorem 5.5 and Theorem 5.6) [1]

(9th Week)
Application: Computer graphics- Fundamental movements in a plane, Homogenous coordinates, Composition of movements
Sections 8.8 [1]

(10th Week)
Kernel and range of a linear transformation, Dimension theorem
Sections 5.3 [1]

(11th Week)
One to one and onto linear transformations, Invertible linear transformations, Isomorphism: Isomorphic vector spaces (to \( \mathbb{R}^n \))
Sections 5.4, 5.5 (Pages 356 to 361, Statements of Theorem 5.14 and Theorem 5.15) [1]

(12th Week)
Orthogonal and orthonormal vectors, Orthogonal and orthonormal bases, Orthogonal complement, Projection theorem (Statement only), Orthogonal projection onto a subspace, Application: Least square solutions for inconsistent systems
Section 6.1 (Pages 397 to 400, Statement of Theorem 6.3), 6.2 (Pages 412 to 418, 422, Statement of Theorem 6.12), 8.12 (Pages 570 to 573, Statement of Theorem 8.12) [1]

REFERENCES:

DIFFERENTIAL EQUATIONS AND MATHEMATICAL MODELING
(SEMESTER V)

Total Marks: 100 (Theory: 60, Practical: 20, Internal Assessment: 20)
4 Lectures + 3 Practical + 1 Presentation

(1st Week)
First order ordinary differential equations: Basic concepts and ideas, Modeling:
Exponential growth and decay, Direction field, Separable equations, Modeling:
Radiocarbon dating, Mixing problem
Sections 1.1, 1.2, 1.3 (Pages 12 to 14) [1]

(2nd Week)
Modeling: Newton's law of cooling, Exact differential equations, Integrating factors,
Bernoulli equations, Modeling: Hormone level in blood, Logistic equation
Sections 1.3 (Pages 14 to 15 and Page 17), 1.4, 1.5 (Pages 29 to 31) [1]

(3rd Week)
Orthogonal trajectories of curves, Existence and uniqueness of solutions, Second order
differential equations: Homogenous linear equations of second order
Sections 1.6, 1.7, 2.1 [1]

(4th Week)
Second order homogenous equations with constant coefficients, Differential operator,
Euler-Cauchy equation
Sections 2.2, 2.3, 2.5 [1]

(5th Week)
Existence and uniqueness theory: Wronskian, Nonhomogenous ordinary differential
equations, Solution by undetermined coefficients
Sections 2.6, 2.7 [1]

(6th Week)
Solution by variation of parameters, Higher order homogenous equations with constant coefficients, System of differential equations, Modeling: Mixing problem involving two tanks

Sections 2.10, 3.2, 4.1(Pages130 to 132) [1]

(7th Week)
System of differential equations: Conversion of \(n^{th}\) order ODEs to a system, Basic concepts and ideas, Homogenous system with constant coefficients, Phase plane, Critical points

Sections 4.1 (Pages 134, 135), 4.2, 4.3 [1]

(8th Week)
Criteria for critical Points and stability, Qualitative methods for nonlinear systems: Linearization of nonlinear systems, Lotka–Volterra population model

Sections 4.4, 4.5 (Pages 151 to 155) [1]

(9th Week)
Power series method: Theory of power series methods, Legendre’s equation, Legendre polynomial

Sections 5.1, 5.2, 5.3 [1]

(10th Week)
Partial differential equations: Basic Concepts and definitions, Mathematical problems, First order equations: Classification, Construction, Geometrical interpretation, Method of characteristics

Sections 2.1, 2.2, 2.3, 2.4 [2]

(11th Week)
General solutions of first order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations

Sections 2.6, 2.7 [2]

(12th Week)
Classification of second order partial differential equations, Reduction to canonical forms, Second order partial differential equations with constant coefficients, General solutions

Sections 4.1, 4.2, 4.3, 4.4 [2]
PRACTICALS

1. To determine whether a given number is prime or composite.
2. To find the sum of digits of a number and decide its divisibility.
3. To compute the roots of a quadratic equation.
4. To Linear Sort a given set of numbers.
5. To compute higher degree polynomials using Horner’s method.
6. To plot the direction field of first order differential equation.
7. To find the solution and plot the growth and decay model (both exponential and logistic).
8. To find the solution and plot the Lotka–Volterra model.
9. To find the solution of Cauchy problem for first order partial differential equations.
10. To plot the integral surfaces of a given first order partial differential equations with initial data.

Note: Programming is to be done in any one of Computer Algebra Systems: MATLAB/MATHEMATICA/MAPLE.

REFERENCES:

FOR PRACTICALS/SUGGESTED READING:
Numerical Methods (Semester VI)

Total Marks: 100 (Theory: 60, Practical: 20, Internal Assessment: 20)
4 Lectures + 3 Practical+1 Presentation
Use of Scientific Calculators is allowed.

(1st Week)
Floating point representation and computer arithmetic, Significant digits, Errors: Round-off error, Local truncation error, Global truncation error, Order of a method, Convergence and terminal conditions, Efficient computations
   Sections 1.2.3, 1.3 (Pages 16 to 25 and Page 30) [1]

(2nd Week)
Bisection method, Secant method, Regula–Falsi method
   Sections 2.1, 2.2 [1]

(3rd Week)
Newton–Raphson method, Newton’s method for solving nonlinear systems
   Sections 2.3, 7.1.1(Pages 266 to 270) [1]

(4th Week)
Gauss elimination method (with row pivoting) and Gauss–Jordan method, Gauss Thomas method for tridiagonal systems
   Sections 3.1 (Pages 110 to 115), 3.2, 3.3 [1]

(5th Week)
Iterative methods: Jacobi and Gauss-Seidel iterative methods
   Sections 6.1 (Pages 223 to 231), 6.2 [1]

(6th Week)
Interpolation: Lagrange’s form and Newton’s form
   Sections 8.1 (Pages 290 to 299 and Pages 304 to 305) [1]

(7th Week)
Finite difference operators, Gregory Newton forward and backward differences
Interpolation
   Sections 4.3, 4.4 (Pages 235 to 236) [2]
(8th Week)
Piecewise polynomial interpolation: Linear interpolation, Cubic spline interpolation (only method), Numerical differentiation: First derivatives and second order derivatives, Richardson extrapolation

Sections 16.1, 16.2 (Pages 361 to 363), 16.4 [3]
Section 11.1 (Pages 426 to 430 and Pages 432 to 433) [1]

(9th Week)
Numerical integration: Trapezoid rule, Simpson’s rule (only method), Newton–Cotes open formulas

Sections 11.2 (Pages 434 to 445) [1]

(10th Week)
Extrapolation methods: Romberg integration, Gaussian quadrature, Ordinary differential equation: Euler’s method

Sections 11.2.4, 11.3.1 [1]
Section 20.2 (Pages 481 to 485) [3]

(11th Week)
Modified Euler’s methods: Heun method and Mid-point method, Runge-Kutta second methods: Heun method without iteration, Mid-point method and Ralston’s method

Sections 20.3, 20.4 (Pages 493 to 495) [3]

(12th Week)
Classical 4th order Runge-Kutta method, Finite difference method for linear ODE

Section 20.4.2 [3]
Section 14.2.1 [1]

PRACTICALS
1. Find the roots of the equation by bisection method (Exercises P2.1 to P2.20 [1])
2. Find the roots of the equation by secant/Regula–Falsi method (Exercises P2.1 to P2.20 [1])
3. Find the roots of the equation by Newton’s method (Exercises P2.11 to 2.29 [1])
4. Find the solution of a system of nonlinear equation using Newton’s method (Exercises P7.1 to P7.15 [1])
5. Find the solution of tridiagonal system using Gauss Thomas method (Exercises P3.21 to P3.25, C3.1 to C3.3, A3.7, A3.8[1])
6. Find the solution of system of equations using Jacobi/Gauss-Seidel method (Exercises P6.1 to P6.18 [1])
7. Find the cubic spline interpolating function (Exercises C8.1 to C8.5 [1])
8. Evaluate the approximate value of finite integrals using Gaussian/Romberg integration (Exercises P11.6 to P11.20 [1])
9. Solve the initial value problem using Euler’s method and compare the result with the exact solutions (Exercises P12.11 to P12.20 [1])
10. Solve the boundary value problem using finite difference method (Exercises P14.1 to P14.25 [1])

Note: Programming is to be done in any one of Computer Algebra Systems: MATLAB/MATHEMATICA/MAPLE.

REFERENCES:
REAL ANALYSIS (SEMESTER VII)

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
4 Lectures, 1 Presentation

(1st Week)
Algebraic and order properties of \( \mathbb{R} \), Positive integers, Statement of well ordering principle, Least upper bound, Greatest lower bound, Completeness property of \( \mathbb{R} \)
   
   Sections 1.3, 1.4 (Page 18), 1.5 (Pages 24 to 26, Statements of Theorem 1.5.10 and Corollary 1.5.11) [1]

(2nd Week)
Archimedean property, Denseness of the sets \( \mathbb{Q} \) and \( \mathbb{Q}^c \) in \( \mathbb{R} \), Sequences, Convergence and divergence of sequence,
   
   Sections 1.5 (From Theorem 1.5.12 onwards), 2.1 [1]

(3rd Week)
Limit theorems, Uniqueness of limit of a sequence, Bounded sequences, Algebra of limits of sequences, Monotonic sequence
   
   Sections 2.2 (Statements of Theorem 2.2.5, Theorem 2.2.7 and Theorem 2.2.9), 2.3 [1]

(4th Week)
Subsequences, Nested interval theorem (without proof), Bolzano Weierstrass theorem, Cluster points
   
   Section 2.5 (Pages 55 to 60) [1]

(5th Week)
Cauchy sequence, Infinite series: Sequence of partial sum, Convergence and divergence, Geometric series, Algebraic theory of series
   
   Sections 2.6, 6.1 (Pages 213 to 218) [1]
(6th Week)
Integral test (without proof), Comparison tests, Ratio test (without proof), Alternating series test, Absolute convergence and conditional convergence
   Sections 6.1 (Pages 219 to 222, Statements of Corollary 6.1.12), 6.2 (Statement of Theorem 6.2.5) [1]

(7th Week)
Illustrations of Taylor’s series & Maclaurin’s series, Taylor’s theorem (without proof)
   Sections 6.4 [1]

(8th Week)
Continuity, Removable discontinuity, Algebra of continuous functions, Sequential criterion of continuity
   Section 3.4 (Statements of Theorem 3.4.11 and Theorem 3.4.16) [1]

(9th Week)
Continuity at end points of [a, b], Intermediate value theorem, Boundedness of a function, Uniform continuity
   Sections 3.5 (Statement of Theorem 3.5.10), 3.6 [1]

(10th Week)
Local maximum, Local minimum, Rolle’s theorem, Mean value theorem, Monotonic function
   Sections 4.3 [1]

(11th Week)
Inverse function, Sequences and series of functions: Pointwise and uniform convergence,
   Section 4.4 (Statements of Theorem 4.4.2 and Theorem 4.4.4), 7.1 (Pages 258 to 265) [1]
(12th Week)
Weierstrass $M$-test (without proof), Consequences of uniform convergence
Section 7.1 (Pages 265 to 267), 7.2 (Statements of Theorem 7.2.3 and Corollary 7.2.11) [1]

REFERENCE:

SUGGESTED READING:
ABSTRACT ALGEBRA (SEMESTER VIII)

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
4 Lectures, 1 Presentation

(1st, 2nd & 3rd Weeks)
Partitions and equivalence relations, Congruence modulo \(n\), Algebra on circles, Roots of unity, Binary operations, Isomorphic Binary Structures, Definition & Elementary Properties of Groups, Finite Groups and Group Tables, Subgroups
   Sections 0, 1 (Pages 29 to 32), 2, 3, 4, 5 [1]

(4th, 5th & 6th Weeks)
Cyclic groups, Cyclic subgroups, Elementary properties of cyclic groups, Subgroups of a finite cyclic group, Groups of permutations, Dihedral group \(D_n\), Cayley theorem, Orbits, Cycles, Even and odd permutations, Alternating groups
   Sections 6 (Statement of Theorem 6.10), Sections 8 (Statement of Lemma 8.15), 9 (Statement of Theorem 9.15) [1]

(7th & 8th Weeks)
Cosets, Lagrange’s theorem, Index of a subgroup, Homomorphism and their properties, Kernel, Isomorphism, Normal subgroup
   Sections 10, 13 (Statement of Theorem 13.15) [1]

(9th & 10th Weeks)
Definition and examples of rings, Basic properties of ring, Field, Division ring, Ring of polynomials, Ring of quaternions
   Sections 18 (Pages 181 to 184, 186 to 188), 22 (Pages 212 to 215), 24 (Pages 238 to 240) [1]

(11th & 12th Weeks)
Integral domain, Characteristic of a ring, Homomorphism, Isomorphism, Ideals
   Section 18 (Pages 185, 186), 19, 26 (Pages 251 to 253, 255) [1]
REFERENCES:

SUGGESTED BOOKS:
The four papers in this category is aimed at providing an opportunity to learn the Mathematics behind the applied courses namely C++ Programming, Mathematical Finance, Cryptography and Networks and Discrete Mathematics. C++ language is used to create computer programs which have applications in the field of systems software, client applications, entertainment software and research. Mathematical finance paper provides the understanding of mathematics behind finance and has wide applications in financial and capital markets. Security is a challenging issue of data communications that the world faces today in the age of computers. Learning effective encryption/decryption methods to enhance data security is the basis of the paper cryptography and networks. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, software development, operations research. Courses are open to all students who have studied mathematics at least till 12th level.

### APPLIED COURSES STRUCTURE

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SEMESTER III

“C++ PROGRAMMING LANGUAGE – INTRODUCTION”

Total marks: 75 Continuous evaluation by the teacher

2 Lectures and 1 Presentation, 2 Practicals (Per week per students)

Introduction to structured programming: data types- simple data types, floating data
types, character data types, string data types, arithmetic operators and operators
precedence, variables and constant declarations, expressions, input using the extraction
operator >> and cin, output using the insertion operator << and cout, preprocessor
directives, increment(++) and decrement(--) operations, creating a C++ program, input/
output, relational operators, logical operators and logical expressions, if and if-else
statement, switch and break statements.

[1] Chapter 2 (pages 37-95), Chapter 3 (pages 96 -129), Chapter 4 (pages 134-178)

“for”, “while” and “do-while” loops and continue statement, nested control statement,
value returning functions, value versus reference parameters, local and global variables,
one dimensional array, two dimensional array, pointer data and pointer variables,.

[1] Chapter 5 (pages 181 - 236), Chapter 6, Chapter 7 (pages 287- 304) Chapter 9
(pages 357 - 390), Chapter 14 (pages 594 - 600).

Reference:
Learning, India Edition

Suggested Readings:
Hill Education Pvt. Ltd.
Education.

Note: Practical programs of the following (and similar) type have to be done.

1. Calculate the Sum of the series 1/1 + 1/2+ 1/3……………….+1/N for any positive
integer N.
2. Write a user defined function to find the absolute value of an integer and use it to evaluate the function \((-1)^n/|n|\), for \(n = -2, -1, 0, 1, 2\).

3. Calculate the factorial of any natural number.

4. Read floating numbers and compute two averages: the average of negative numbers and the average of positive numbers.

5. Write a program that prompts the user to input a positive integer. It should then output a message indicating whether the number is a prime number.

6. Write a program that prompts the user to input the value of \(a\), \(b\) and \(c\) involved in the equation \(ax^2 + bx + c = 0\) and outputs the type of the roots of the equation. Also the program should outputs all the roots of the equation.

7. Write a program that generates random integer between 0 and 99. Given that first two Fibonacci numbers are 0 and 1, generate all Fibonacci numbers less than or equal to generated number.

8. Write a program that does the following:
   a. Prompts the user to input five decimal numbers.
   b. Prints the five decimal numbers.
   c. Converts each decimal number to the nearest integer.
   d. Adds these five integers.
   e. Prints the sum and average of them.

9. Write a program that uses \texttt{while} loops to perform the following steps:
   a. Prompt the user to input two integers: \texttt{firstNum} and \texttt{secondNum} (\texttt{firstNum} should be less than \texttt{secondNum}).
   b. Output all odd and even numbers between \texttt{firstNum} and \texttt{secondNum}.
   c. Output the sum of all even numbers between \texttt{firstNum} and \texttt{secondNum}.
   d. Output the sum of the square of the odd numbers \texttt{firstNum} and \texttt{secondNum}.
   e. Output all uppercase letters corresponding to the numbers between \texttt{firstNum} and \texttt{secondNum}, if any.

10. Write a program that prompts the user to input five decimal numbers. The program should then add the five decimal numbers, convert the sum to the nearest integer, and print the result.

11. Write a program that prompts the user to enter the lengths of three sides of a triangle and then outputs a message indicating whether the triangle is a right triangle or a scalene triangle.

12. Write a value returning function \texttt{smaller} to determine the smallest number from a set of numbers. Use this function to determine the smallest number from a set of 10 numbers.
13. Write a function that takes as a parameter an integer (as a long value) and returns the number of odd, even, and zero digits. Also write a program to test your function.

14. Enter 100 integers into an array and sort them in an ascending/descending order and print the largest/smallest integers.

15. Enter 10 integers into an array and then search for a particular integer in the array.


17. Using arrays, read the vectors of the following type: \( A = (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) \), \( B = (0 \ 2 \ 3 \ 4 \ 0 \ 1 \ 5 \ 6 \) \) and compute the product and addition of these vectors.

18. Read from a text file and write to a text file.

19. Write a program to create the following grid using for loops:

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 & 5 \\
2 & 3 & 4 & 5 & 6 \\
3 & 4 & 5 & 6 & 7 \\
4 & 5 & 6 & 7 & 8 \\
5 & 6 & 7 & 8 & 9 \\
\end{array}
\]

20. Write a function, \( reverseDigit \), that takes an integer as a parameter and returns the number with its digits reversed. For example, the value of function \( reverseDigit(12345) \) is 54321 and the value of \( reverseDigit(-532) \) is -235.

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Week Wise Distribution

[ I\textsuperscript{st} week ]

Simple data types, Floating data types, Character data types, String data types. Arithmetic operators and operators precedence

[ II\textsuperscript{nd} Week ]

Variables and constant declarations. Expressions

[ III\textsuperscript{rd} Week ]

Input using the extraction operator \( >> \) and cin. Output using the insertion operator \( << \) and cout.

[ IV\textsuperscript{th} Week ]

Preprocessor directives, Increment(++) and decrement operations(--)}. Creating a C++ program
[ Vth Week ]
Input/ Output. Relational operators, Logical operators and logical expressions

[ VIth Week ]
if and if-else statement.

[ VIIth Week ]
switch and break statements. “for” statement.

[ VIIIth Week ]
“while” and “do-while” loops and continue statement

[ IXth Week ]
Nested control statement, Value returning functions.

[ Xth Week ]
Void functions, Value versus reference parameters.

[ XIth Week ]
One dimensional array

[ XIIth Week ]
Two dimensional array, Pointer data and pointer variables.
SEMESTER IV
MATHEMATICAL FINANCE

Total marks: 75 Continuous evaluation by the teacher
2 Lectures and 1 Presentation, 2 Practicals (Per week per students)

Week-1
Interest rates, types of rates, measuring interest rates, zero rates, bond pricing
[1] Chapter 4 (4.1-4.4)

Week-2
Forward rate, duration, convexity
[1] Chapter 4 (4.6, 4.8-4.9)

Week-3
Exchange Traded Markets and OTC markets, Derivatives- Forward contracts, futures contract, options,
Types of traders, hedging, speculation, arbitrage
[1] Chapter 1 (1.1-1.9)

Week-4
No Arbitrage principle, short selling, forward price for an investment asset
[1] Chapter 5 (5.2-5.4)

Week-5
Types of Options, Option positions, Underlying assets, Factors affecting option prices
[1] Chapter 8 (8.1-8.3), Chapter 9 (9.1)

Week-6
Bounds on option prices, put-call parity, early exercise, effect of dividends
[1] Chapter 9 (9.2-9.7)
Week-7

Binomial option pricing model, Risk neutral Valuation (for European and American options on assets following binomial tree model)

[1] Chapter 11(11.1-11.5)

Week-8

Lognormal property of stock prices, distribution of rate of return, expected return, volatility, estimating volatility from historical data


Week-10

Extension of risk neutral valuation to assets following GBM (without proof), Black Scholes formula for European options

[1] Chapter 13 (13.7-13.8)

Week-11

Hedging parameters (the Greeks: delta, gamma, theta, rho and Vega)

[1] Chapter 17 (17.1-17.9)

Week-12

Trading strategies Involving options

[1] Chapter 10 (except box spreads, calendar spreads and diagonal spreads)

Week-12

Swaps, mechanics of interest rate swaps, comparative advantage argument, valuation of interest rate swaps, currency swaps, valuation of currency swaps

[1] Chapter 7 (7.1-7.4, 7.7-7.9)

Reference

**For further reading**


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**Practical/Lab work using Excel**

3. Bond duration and convexity.
5. Estimating volatility from historical data of stock prices.
6. Simulating a binomial price path.
7. Computing price of simple European call and put options when the underlying follows binomial model (using Monte Carlo simulation).
8. Simulating lognormal price path.
9. Computing price of simple European call and put options when the underlying follows lognormal model (using Monte Carlo simulation).
10. Implementing the Black-Scholes formulae in a spreadsheet.
12. Valuation of a swap.

**References for Practicals:**

Definition of a cryptosystem, Symmetric cipher model, Classical encryption techniques-Substitution and transposition ciphers, caesar cipher, Playfair cipher.

[1] 2.1-2.3

Block cipher Principles, Shannon theory of diffusion and confusion, Data encryption standard (DES).

[1] 3.1, 3.2, 3.3.

Polynomial and modular arithmetic, Introduction to finite field of the form GF(p) and GF(2^n), Fermat theorem and Euler’s theorem(statement only), Chinese Remainder theorem, Discrete logarithm.

[1] 4.2, 4.3, 4.5, 4.6, 4.7, 8.2, 8.4, 8.5

Advanced Encryption Standard(AES), Stream ciphers . Introduction to public key cryptography, RSA algorithm and security of RSA, Introduction to elliptic curve cryptography.

[1] 5.2-5.5(tables 5.5, 5.6 excluded),7.4, 9.1, 9.2, 10.3, 10.4

Information/Computer Security: Basic security objectives, security attacks, security services, Network security model,

[1]1.1, 1.3, 1.4, 1.6

Cryptographic Hash functions, Secure Hash algorithm, SHA-3.
[1] 11.1, 11.5, 11.6
Digital signature, Elgamal signature, Digital signature standards, Digital signature algorithm


E-mail security: Pretty Good Privacy (PGP)

[1] 18.1 Page 592-596(Confidentiality excluded)

REFERENCE:

SUGGESTED READING:

Week Wise Distribution

[ I
\(^{st}\) week ]
Definition of a cryptosystem, Symmetric cipher model, Classical encryption techniques-Substitution and transposition ciphers, caesar cipher, Playfair cipher.

[ II
\(^{nd}\) Week ]
Block cipher Principles, Shannon theory of diffusion and confusion.

[ III
\(^{rd}\) Week ]
Data encryption standard (DES). Polynomial and modular arithmetic, Introduction to finite field of the form GF(p) and GF(2^n).

[ IV
\(^{th}\) Week ]
Fermat theorem and Euler’s theorem(statement only), Chinese Remainder theorem, Discrete logarithm.
[ Vth Week ]
Advanced Encryption Standard (AES), Stream ciphers.

[ VIth Week ]
Introduction to public key cryptography, RSA algorithm and security of RSA.

[ VIIth Week ]
Introduction to elliptic curve cryptography.

[ VIIith Week ]
Basic security objectives, security attacks, security services, Network security model.

[ IXth Week ]
Cryptographic Hash functions, Secure Hash algorithm, SHA-3.

[ Xth Week ]
Digital signature, Elgamal signature.

[ XIth Week ]
Digital signature standards, Digital signature algorithm.

[ XIIth Week ]
Pretty Good Privacy (PGP).
SEMESTER VI
DISCRETE MATHEMATICS

Total marks: 75 Continuous evaluation by the teacher
2 Lectures and 1 Presentation

Week 1:
Definition, examples and properties of posets, maps between posets.

Week 2:
Algebraic lattice, lattice as a poset, duality principal, sublattice, Hasse diagram.

Week 3:
Products and homomorphisms of lattices, Distributive lattice, complemented lattice.

References for first 3 weeks:
[1] Chapter 1 (Section 1, Section 2 - upto Theorem 2.9)

Week 4:
Boolean Algebra, Boolean polynomial, CN form, DN form.

Week 5:
Simplification of Boolean polynomials, Karnaugh diagram.

Week 6:
Switching Circuits and its applications.

References for Week 4, 5 and 6:
[1] Chapter 1 (Section 3 – Upto example 3.9), (Section 4- upto Definition 4.8, Finding CN form and DN form as in example 4.16 and 4.17, Section 6 (page 48 to 50), section 7, section 8- example 8.1)
Week 7:
Graphs, subgraph, complete graph, bipartite graph, degree sequence, Euler’s theorem for sum of degrees of all vertices.

Week 8:
Eulerian circuit, Seven bridge problem, Hamiltonian cycle, Adjacency matrix.

Week 9:
Dijkstra’s shortest path algorithm (improved version).

References for week 7, 8 and 9:
[2] Chapter 9 (9.2), Chapter 10 (10.1, 10.2 – 10.2.1, 10.2.2 and application to Gray codes, 10.3, 10.4)

Week 10:
Chinese postman problem, Digraphs.

Week 11:
Definitions and examples of tree and spanning tree, Kruskal’s algorithm to find the minimum spanning tree.

Week 12:
Planar graphs, coloring of a graph and chromatic number.

References for Week 10, 11 and 12:
[2] Chapter 11 (11.1, 11.2- upto 11.2.4)
   Chapter 12 (12.1, 12.2, 12.3)
   Chapter 13 (13.1- upto 13.1.6, 13.2)

References:

Suggested Reading: